Embedded Flow Characteristics of Sharp-Edged Rectangular Wings

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Abstract

An exploratory investigation into the construction of semiempirical, analytic expressions for steady, symmetric flow characteristics on thin rectangular wings at transonic speed is presented. Early contributions from the small-disturbance transonic flow theory are used as a guide. These solutions for two-dimensional wings with rombic sections, extending over most of the supersonic portion of the transonic region, are represented by analytic expressions obtained through modification of corresponding expressions prepared for subsonic flow. The result is applied on a wing with an aspect ratio of 2 and three thickness-to-chord ratios. Experiments for comparison are not available.

Contents

Solutions in Refs. 1-3 for the incremental lift at zero angle of attack, K_p, and for the position of lift along the wing chord, $(x/c)_p$, taken from Ref. 3 (Fig. 105, p. 290), are shown in Fig. 1. The variables on the vertical axes are either a thickness parameter,

$$\rho = [(\gamma+1)\theta]^{1/3} \tag{1}$$

multiplied by K_p or the c.p. value, $(x/c)_p$. θ and γ are, respectively, half the profile apex angle and the ratio of specific heats.

The variable on the horizontal axis is the transonic similarity parameter

$$t = \eta\theta^{-2/3} = \frac{M_\infty - 1}{\frac{1}{2}[(\gamma+1)\theta]^{2/3}} = \frac{M_\infty - 1}{\frac{1}{2}\rho^2} \tag{2}$$

where η is Pistolesi's hodograph variable.[3]

Two-Dimensional Flow, $M_\infty \geq 1$

As a lead for the determination of semiempirical expressions representing the theoretical solutions in Fig. 1, the K coefficients,[4] closely connected with solutions of linearized theory at $M_\infty \leq 1$, are used:

$$\beta K_p = \frac{2\pi\beta A}{2+\sqrt{(4/3)(\beta A)^2+4}} \tag{3}$$

$$\beta K_{v,\mathrm{le}} = \frac{\pi\beta A}{2+\sqrt{(1/4)(\beta A)^2+4}} \tag{4}$$

$$K_{v,\mathrm{se}} = \frac{2\pi}{2+\beta A} \tag{5}$$

where $K_{v,\mathrm{le}}$ and $K_{v,\mathrm{se}}$ are the leading-edge and the total side-edge suction force coefficients, respectively. A is the aspect ratio and $\beta = \sqrt{1-M_\infty^2}$. Equations (3-5) are of the same basic type but, at present, only Eq. (3) can be used directly because relevant results for a similar usage of Eq. (4) are not available and Eq. (5) becomes zero when A goes to infinity.

After replacing β with ρ, and inserting new empirically determined constants, and completing with a function $f(t)$ under the square root signs, a set of new K coefficients is obtained. They are specialized to two-dimensional sharp-edged rectangular wings in a so-called embedded subsonic flow and are

$$\rho K_p = \frac{2\pi}{\sqrt{(9/10)f(t)}} \tag{6}$$

$$\rho K_{v,\mathrm{le}} = \frac{\pi}{\sqrt{(1/6)f(t)}} \tag{7}$$

$$K_{v,\mathrm{se}} = 0 \tag{8}$$

The ratio between the constants under the square root signs has been retained from Eqs. (3) and (4). The function $f(t)$ has been empirically determined in the interval $0 \leq t \leq 1.26$ from the solutions shown in Fig. 1, i.e., up to shock attachment with pure supersonic flow. The function $f(t)$ is

$$f(t) = 4 - t(t-0.45)[g_1(t)+g_2(t)] \tag{9}$$

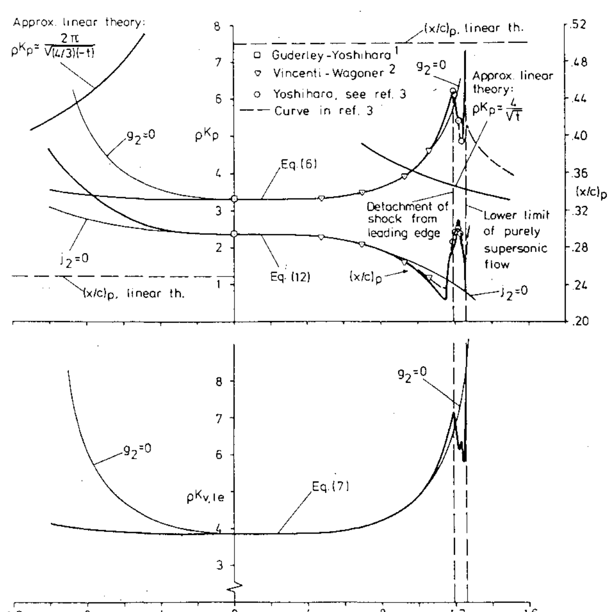

Fig. 1 Analytic representations of the solutions in Refs. 1-3 by Eqs. (6) and (12), and the semiempirical approximation, Eq. (7), of the generalized $\rho K_{v,\mathrm{le}}$.

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where

$$g_1(t) = 2.74t^{6/11} \tag{10}$$

and

$$g_2(t) = \frac{5(t-1)\,|1.2-t|^{3/11}(t-1.26)^{2/11}}{(t-1.2)^{0.4}+3(t-1.245)^{6/11}}\,\mathrm{sgn}(1.2-t) \tag{11}$$

At $t=1$, $M_\infty=1$, and Eq. (6) reduces to 3.32, which is practically the same result as in Fig. 1. In the same manner Eq. (7) reduces to 3.83, which can at present be related only to $\beta K_{v,\mathrm{le}}=2\pi$, i.e. $K_{v,\mathrm{le}}\to\infty$ according to linearized theory, Eq. (4).

The center of pressure is approximated by the expression

$$\left(\frac{x}{c}\right)_p = \frac{\sqrt{(6/7)h(t)}}{2\pi} = \frac{1}{2\pi\sqrt{(7/6)\,[1/h(t)]}} \tag{12}$$

where the empirical function is

$$h(t) = 4 - t^2[j_1(t) + j_2(t)] \tag{13}$$

with

$$j_1(t) = 0.85t^{5/9} \tag{14}$$

and

$$j_2(t) = \frac{3.4(t-0.85)\,|1.16-t|^{3/11}(t-1.26)^{2/11}}{(t-1.16)^{0.4}+2(t-1.22)^{6/11}}$$

$$\times t^{12/11}\,\mathrm{sgn}(1.16-t) \tag{15}$$

Three-Dimensional Flow, $M_\infty \geq 1$

The K coefficients on three-dimensional wings for $0\leq t\leq 1.26$, expressed in the t and ρ variables, are approximated by

$$\rho K_p = \frac{2\pi\rho A}{2+\sqrt{(9/10)f(t)\,(\rho A)^2+4}} \tag{16}$$

$$\rho K_{v,\mathrm{le}} = \frac{\pi\rho A}{2+\sqrt{(1/6)f(t)\,(\rho A)^2+4}} \tag{17}$$

$$K_{v,\mathrm{se}} = \frac{2\pi}{2+\sqrt{(t/2)\,(M+1)\rho A}} \cong \frac{2\pi}{2+\sqrt{t\rho A}} \tag{18}$$

The last equation is obtained by only a formalistic change of β in Eq. (5) to $|\beta| = \rho\sqrt{(t/2)(M+1)}$ by use of Eq. (2) and is, therefore, an extreme approximation.

For the centers of pressure of the different partial lift contributions, the following expressions are quite good approximations:

$$\left(\frac{x}{c}\right)_p = \frac{A}{2\pi\sqrt{(7/6)\,(1/h(t))A^2+1}} \tag{19}$$

$$\left(\frac{x}{c}\right)_{v,\mathrm{le}} = 0 \tag{20}$$

$$\left(\frac{x}{c}\right)_{v,\mathrm{se}} = \frac{1}{2} \tag{21}$$

Equations (20) and (21) are "first approximations" only.

Results

The guiding solutions[1,2] for the generalized ρK_p and $(x/c)_p$, as seen in Fig. 1, are better represented in the interval $0\leq t<1$ by Eqs. (6), (9-11), and (12-15), respectively, than for $1<t\leq 1.26$. Good representations of the theoretical ρK_p and $(x/c)_p$ are obtained in the interval $0\leq t<1$ and $0\leq t<0.8$, respectively, without the terms g_2 and j_2, which are needed only to describe the major flow changes that take place in the interval $1<t\leq 1.26$. An extrapolation into the subsonic region, $t<0$, has been done. It is believed that the steeper curves come closest to a higher-order theoretical solution when $M_\infty<1$. The generalized leading-edge suction force coefficient, $\rho K_{v,\mathrm{le}}$ is inserted in Fig. 1. A theoretical solution on this coefficient seems not to have been derived yet.

Equations (16-18) and (19), are evaluated for a wing with a rombic section and with $A=2$ and $\theta=0.002$, 0.01, and 0.06. The results are plotted in Fig. 2. The large influence of the thickness on K_p and $(x/c)_p$ is contrasted to the minor or nonexistent thickness dependence in $K_{v,\mathrm{le}}$ and $K_{v,\mathrm{se}}$, respectively. As for K_p, it is evident that the gap at $M_\infty\sim 1$ between the curves obtained by linearized theory would be more or less filled out when $\theta\to\epsilon>0$.

Solutions from the full potential flow equation are needed in order to decide the quality of the present semiempirical representations of the K coefficients and the c.p. locations on wings with finite aspect ratio and rombic profiles.

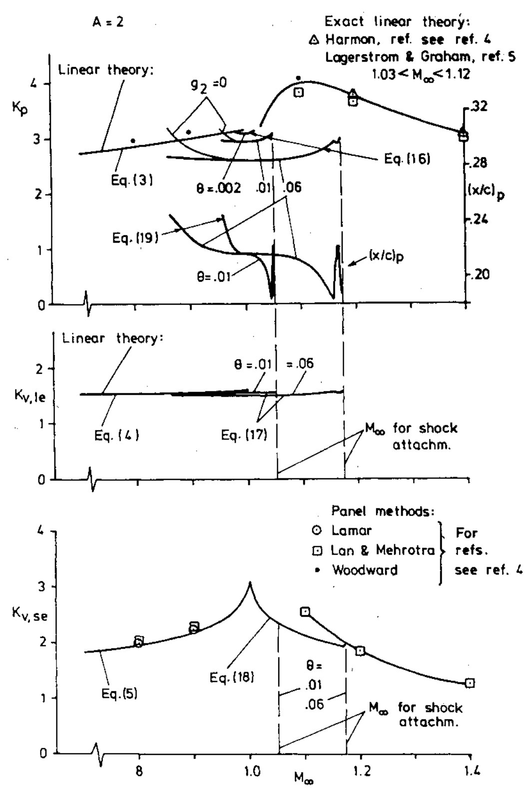

Fig. 2 The semiempirical, embedded symmetric flow characteristics, K_p, $K_{v,\mathrm{le}}$, and $K_{v,\mathrm{se}}$ and $(x/c)_p$ vs M, obtained by Eqs. (16-19), on a rectangular wing with rombic section and $A=2$ and $\theta=0.002$, 0.01, and 0.06. Comparison with linearized theory.

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